Notes.

Course- Diploma
Subject- Applied Maths.

UNIT-1

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1. Law of Indices :-

★ **Introduction** :- A power or an index is used to write product of numbers very compactly. The Plural of Index is Indices.

★ **Power or Indices** :- we write the expression $3 \times 3 \times 3 \times 3$ as 3^4.

we read this as "three to the power four or three raise to power four".
In the expression b^c, b is called the base & c is called the Index.

⇒ **Rules or Law of Indices** :-

Rule-1 $a^m \times a^n = a^{m+n}$

Ex - $2^5 \times 2^3 = 2^8$

Rule-2 $\frac{a^m}{a^n} = a^{m-n}$

Ex - $\frac{5^7}{5^3} = 5^{7-3} = 5^4$

Rule-3 - $(a^m)^n = a^{mn}$

Ex - $(10^3)^7 = 10^{21}$

(ii). If $x+iy=0$ then $x=0$ & $y=0$ i.e. If a complex number is zero then its real part & imaginary Part both are zero.

⇒ **Conjugate of a complex Number.**

If $z=x+iy$ is a complex number than conjugate of z is $x-iy$. It is denoted by $\bar{z}$

$$\therefore \quad \bar{z}=x-iy$$

For example -

If $z=2+3i$, then conjugate of complex number.

Properties of conjugate of complex number

(i) $\overline{(\bar{z})}=z$

(ii) $\left(\overline{z_1+z_2}\right)=\bar{z}_1+\bar{z}_2$

(iii) $\overline{z_1 z_2}=\bar{z}_1\bar{z}_2$

(vi) $i^5 = i^4 . i = i$

(vii) $i^{25} = (i^4)^6 . i = 1 \times i = i$

(viii) $i^{4n+1} = (i^4)^n . i = 1 \times i = i$

Real & Imaginary part of complex number :-

A number of the form $x+iy$, where x & y are real numbers & i is an imaginary number with property $i^2 = -1$ i.e. $i = \sqrt{-1}$ is called a complex number. The complex number is denoted by z

$$z = x + iy$$

The complex number $z = x+iy$ can be re-presented as

$$z = \text{real Part} + i\ (\text{imaginary Part})$$

Properties of complex Numbers :-

complex number;

(i) Equality of two complex number;

Let $z_1 = x_1 + iy_1$ & $z_2 = x_2 + iy_2$ are two complex numbers,

If $z_1 = z_2 \Rightarrow x_1 + iy_1 = x_2 + iy_2$

Then $x_1 = x_2$ & $y_1 = y_2$

i.e. their real & imaginary parts are separately equal.

then it is called improper fraction.

For example ;- $\frac{x^3-5}{x^2-7x+12}$.

Complex Numbers

★ **Number system** – we know the number system as.

1). Natural numbers, $N = \{1, 2, 3, ----\}$

2). whole Numbers, $W = \{0, 1, 2, 3, ---\}$

3). Integers, $Z = \{-3, -2, -1, 0, 1, 2, 3 ---\}$

4). Rational numbers, $Q = \{\frac{p}{q}, p, q \in Z, q \neq 0\}$

5). Irrational numbers -

The numbers whose decimal representation is non-terminating & non repeating.

e.g - $\sqrt{2}, \sqrt{3}$ etc.

6). Real numbers (R) = (Rational numbers + Irrational numbers)

Powers of iota (i)

(i). $i = \sqrt{-1}$

(ii) $i^2 = 1$

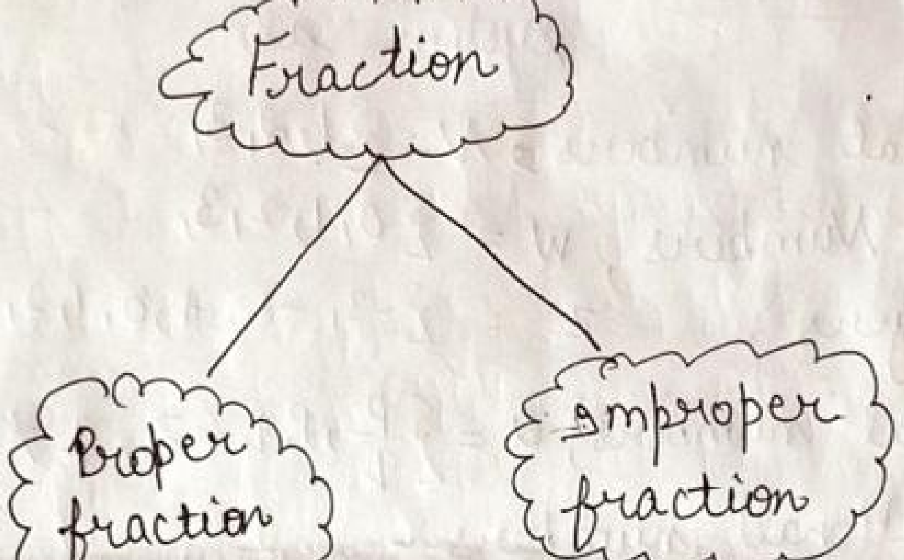
(iii) $i^3 = i^2 . i = (-1) i = i$

(iv) $i^4 = (i^2)^2 = (-1)^2 = 1$

Rational Fraction :-

An expression of the type $\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials & $q(x) \neq 0$ is known as rational fraction.

For example - $\frac{2x+5}{x^2-5x+4}$, $\frac{1}{x^2-1}$



1). Proper fraction :-

If degree of numerator is lower than degree of denominator, it is called proper fraction.

For example $\frac{2x+1}{(2x-1)(x+2)}$

2). Improper fraction :-

If the degree of numerator a greater or equal to the degree of denominator,

☆ Partial Fraction ☆

Fraction - An expression of the form $\frac{p}{q}$, where p & q are integers & $q \neq 0$ is known as a fraction.

Polynomial - An expression of the type

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_n$$

where $a_0, a_1, \ldots a_n$ are constants, is called polynomial.

For example - (i) $x^3 + 2x^2 + 7x + 2$

(ii) $4x^4 + 7x^3 + 9x^2 + 3$

Degree of Polynomial :- Degree of polynomial is the power of heighest term in x (variable).

For ex - Degree is 3.

Polynomial with different degree's

Name	Degree	Examples.
Constant	zero	$7, 9, 11/2$ etc
Linear	1	$x+1, x-3, 5x-3$ etc.
quadratic	2	$x^2+7x+10, 3x^2+7x+11$
cubic	3	x^3+2x^2+7x+2

⇒ Formulae of Algebra

For any two numbers a & b.

1) Square of a sum

$(a+b)^2 = a^2 + 2ab + b^2$

2). Square of a difference.

$(a-b)^2 = a^2 - 2ab + b^2$

3). Difference of two squares.

$(a^2 - b^2) = (a+b)(a-b)$

4) Difference of two cubes.

$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

5) Sum of two cubes.

$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$

6). Cube of a sum

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

7). Cube of a difference

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Factorization formula / Quadratic Formula

If a, b & c are real numbers, then $ax^2 + bx + c = 0$ has solution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rule-4 - $a^0 = 1$

Ex - $10^0 = 1$

Rule-5 - $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Ex - $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

Rule-6 - $(ab)^n = a^n b^n$

Ex - $(2a)^5 = 2^5 a^5 = 32a^5$

Rule-7 - $a^{-m} = \frac{1}{a^m}$

Ex - $(9)^{-2} = \frac{1}{9^2} = \frac{1}{81}$

Rule-8 - $a^{n/m} = \sqrt[m]{a^n}$

Ex - $8^{2/3} = \sqrt[3]{8^2} = (8)^{2/3} = (2^3)^{2/3}$
$= 2^2 = 4$

Rule-9 - $a^m = b^m \Rightarrow a = b$
(If the powers are equal then bases are equal)

Ex - $a^5 = b^5 \Rightarrow a = b$

Rule-10 - If $a^m = a^n \Rightarrow m = n$

Rule-11 - $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab} = (ab)^{1/n}$